OPTIMAL CONTROL IN THE PROBLEM OF UNPACKING A SPACE TELESCOPE MIRROR

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The problem of optimal control in the processes of unpacking a space telescope-detector is considered. The telescope consists of a number of separate hexagonal segments and is designed to observe the Earth's atmosphere. At each stage of unpacking, the telescope is regarded as a system of two absolutely rigid bodies (one is packed and the other is unpacked) linked together by a cylindrical hinge. The controlling moment in the hinge is caused by an electric motor. It is required to determine the control (in other words, a level of voltage impressed on the motor) that minimizes the energy spent for unpacking the telescope.

Let us consider a telescope designed to observe processes in the Earth's atmosphere. Its mirror is directed toward the Earth [1, 2]. It is assumed that a space station, where the telescope is installed, moves along a circular orbit. The mirror consists of a number of regular hexagonal segments (Figure 1). Since the mirror is transported to the orbit, the problem of unpacking the mirror arises. The dotted line in Figure 1 illustrates the sequence of unpacking the segments.

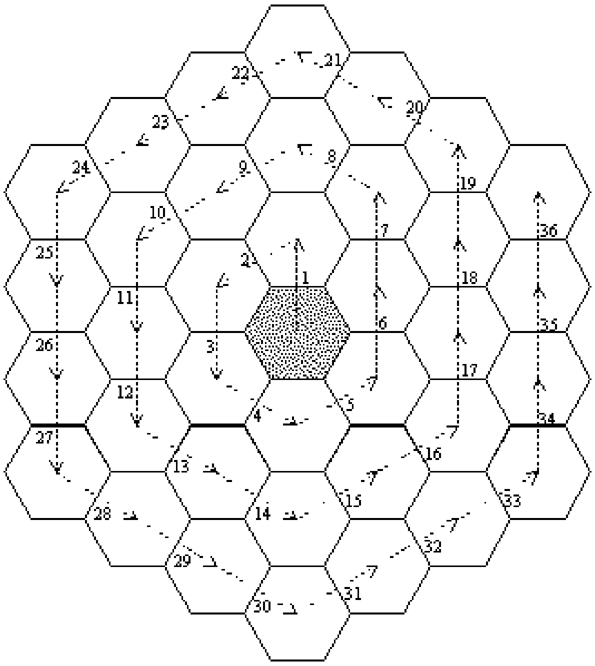

Fig. 1

In the packed state, all the segments are linked together in such a way that the entire configuration can be considered as an absolutely rigid body. The process of unpacking progresses sequentially relative to the initial (zero) segment. The segments are fastened as follows: any intermediate segment is linked only with the two adjacent ones by cylindrical hinges. Thus, we shall assume that the edge of a segment coincides with the corresponding edge of another one. Figure 2 demonstrates the unpacking of three segments.

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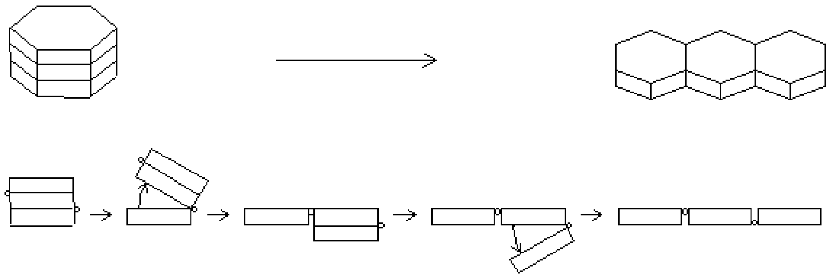

Fig. 2

It is assumed that the unpacked part of the mirror is associated rigidly with the station that moves along the orbit in such a manner that the unpacked part is directed toward the Earth all the time.

Let us introduce a coordinate system $EXYZ$ with its origin E at the center of the Earth. The axes EX and EZ are positioned on the plane of the orbit, whereas the axis EY is directed along the vector $\boldsymbol{\omega}_0$ of the absolute angular velocity of the station. This coordinate system is supposed to be inertial. Let us introduce another coordinate system $Ex^s y^s z^s$ such that it rotates relative to the system $EXYZ$ with the angular velocity $\boldsymbol{\omega}_0 = \omega_0 \boldsymbol{e}_\gamma$ in such a way that the axis Ex^s passes through the center S of mass of the orbital station (Figure 3).

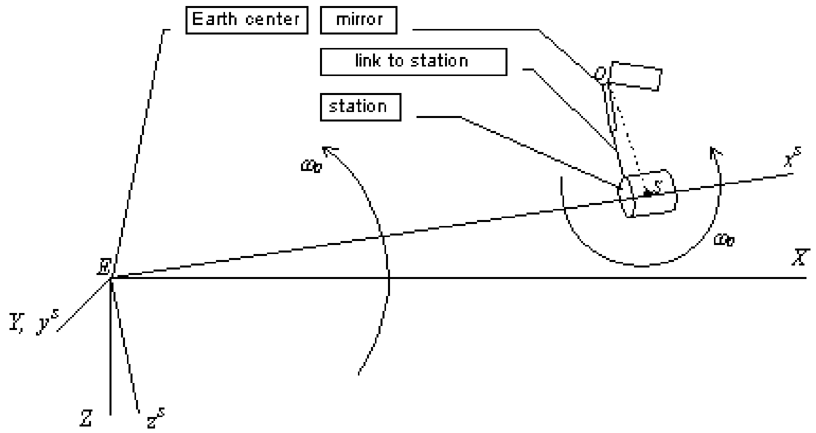

Fig. 3

The coordinate system $Ox_2y_2z_2$ is obtained by the translation of the system $Ex^s y^s z^s$ at the midpoint O of the hinge axis along which the unpacking of the next segment takes place (note that $Ox_2 \parallel Ex^s$, $Oy_2 \parallel Ey^s$, and $Oz_2 \parallel Ez^s$). The coordinate system $Ox_1y_1z_1$ is obtained by the rotation of $Ox_2y_2z_2$ about the axis Ox_2 by an angle α (this angle is constant for each stage of unpacking). The coordinate system $Oxyz$ associated rigidly with the segments is obtained by the rotation of the system $Ox_1y_1z_1$ about the axis Oz_1 by an angle φ. The point L is the center of mass for the unpacked part of the mirror (Figure 4).

Assuming that the Earth's gravitational field is central, we obtain the following equation describing the motion in the coordinate system $Ox_1y_1z_1$:

$$C\ddot{\varphi} = G(\varphi) + M_{\text{mot}} + M_{\text{fr}}$$

$$G(\varphi) = m\omega_0^2 l\big(-3\xi\cos(\varphi+\gamma) + \sin(\zeta\cos\alpha + \eta\sin\alpha)\sin(\varphi+\gamma) + 0.5\,l\cos^2\alpha\sin 2(\varphi+\gamma)\big)$$

$$\varphi = \varphi(t), \quad \varphi(0) = 0, \quad \varphi(t_1) = \pi, \quad \dot{\varphi}(0) = 0, \quad \dot{\varphi}(t_1) = 0$$

Here C is the moment of inertia of the unpacked part of the mirror relative to the hinge axis, m is the mass of the unpacked part, l is the distance between the center of mass of the unpacked part and the hinge axis, M_{fr} is the moment of friction in the hinge, M_{mot} is the moment caused in the hinge by the electric motor, ξ, η and ζ are the coordinates of the point S in the system $Ox_1y_1z_1$: $\boldsymbol{OS} = \xi\boldsymbol{e}_{x_1} + \eta\boldsymbol{e}_{y_1} + \zeta\boldsymbol{e}_{z_1}$.

Further we assume that $\dot{\varphi}$ is proportional to the angular velocity of rotation of the motor's rotor with the coefficient of proportionality j. As a model of the electric motor, we adopt the equation

$$L\dot{I} + RI + \varepsilon j\varphi = k_{\text{g}}U, \quad M_{\text{mot}} = aI$$

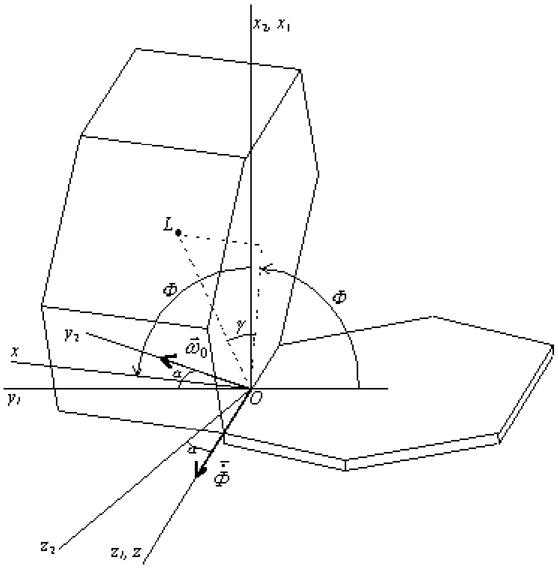

Fig. 4

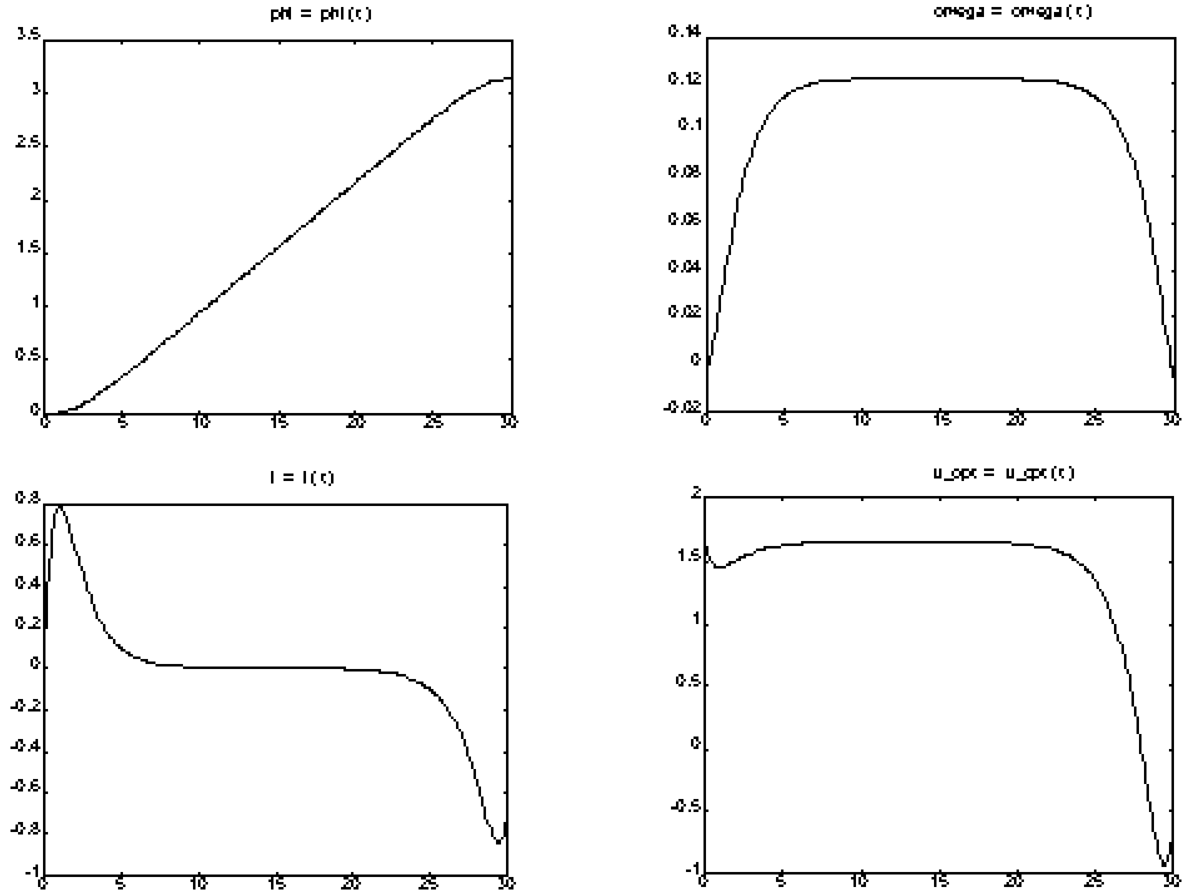

Fig. 5

where I is the current intensity in the coil of the motor moving armature, L is the inductance of the coil, R is the resistance of the armature, ε is the coefficient of the back electromotive force, j is the gear ratio, k_g is the voltage gain, a is the momentum characteristic of the motor, and U is the external voltage considered as a control.

The optimization of the process of unpacking the telescope mirror is regarded as finding a law for the formation of the voltage U such that the functional

$$J_0 = \frac{1}{2} \int_0^{t_1} \left(k_1 I^2 + k_2 U^2\right) d\tau \to \min_U$$

reaches its minimal value with a fixed terminal time t_1 and in the absence of any restrictions imposed on U.

The physical meaning of this approach consists in the following. First, we minimize the integral of the mechanical moment produced by the motor:

$$\int_0^{t_1} k_1 I^2 \, d\tau = \frac{k_1}{a^2} \int_0^{t_1} M_{\mathrm{mot}}^2 \, d\tau$$

Second, we minimize the energy spent for the unpacking of the next segment:

$$\int_0^{t_1} k_2 U^2 \, d\tau$$

Using Pontrjagin's maximum principle, we reduce this optimization problem to a two-point boundary value problem in the seven-dimensional space.

The MATLAB program was used to solve the latter problem. Some of numerical results we obtained are presented in Figure 5.

The question of finding maximal errors arising in the course of unpacking under the programmed control (given as a time function) is of particular interest. We restrict ourselves to the consideration of two possible sources of errors: the inaccuracy in initial conditions and the ignoring of quantities of higher order in smallness that enter into the equations describing the process of unpacking. Assuming the above errors small and considering the linear terms only, we obtain the linear second-order error equation with variable coefficients:

$$\ddot{x} + f_0 \dot{x} + f_1 x = f_2$$

Here $x(t) = \varphi(t) - \varphi^{\mathrm{com}}(t)$ is the error, $\varphi^{\mathrm{com}}(t)$ is the computed angle of rotation, $f_0 = \mathrm{const}$, $|f_1(t)| \leqslant f_1^* = \mathrm{const}$, and $|f_2(t)| \leqslant f_2^* = \mathrm{const}$. Next we solve the problem of finding a maximal value of $y(t_1) = b^T x(t_1)$, where $b^T = (1, 0, 0)$ or $b^T = (-1, 0, 0)$ with $|x(0)| \leqslant x_0$, $|\dot{x}(0)| \leqslant \dot{x}_0$.

The above results can be used to construct space telescopes for the registration of high energy particles.

REFERENCES

1. V.V. Alexandrov et al., *Vestnik MSU. Astronomy. Physics*, 6: 33, 2000.
2. V.V. Alexandrov et al., *Proc. of Workshop on Observing of Extremely High Energy Cosmic Rays from Earth and Space, AIP Conf. Proc.*, Woodbury, New York, 2001, in press.