AN INVERSE PROBLEM OF ELECTROENCEPHALOGRAPHY FOR A MODEL OF HUMAN BRAIN ACTIVITY

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A human head is modeled by two embedded nonconcentric spheres filled with conducting mediums. The skull is regarded as the outer sphere; the brain is situated in the inner sphere. The inverse problem under study consists in determining the current sources in the celebral cortex with the use of measurements of the electric potential registered on the surface of the skull. It is assumed that the electric potential satisfies the Laplace equation with some boundary conditions. A solution to this boundary value problem is sought as a combination of spherical functions with taking into account the distance between the centers of the spheres. A computational scheme for determining the currents on the basis of electroencephalographic measurements is proposed.

A human head is modeled by two embedded nonconcentric spheres S_1 and S_2 filled with mediums of conductivities σ_1 and σ_2. The skull is regarded as the outer sphere S_2 of radius b. It is assumed that the brain is situated within the inner sphere S_1 of radius a. By ε we denote the distance between the centers of the spheres S_1 and S_2.

Based on the Maxwell equations for the quasistationary case, we formulate the following boundary value problem for the electric potential u:

$$\nabla^2 u = 0 \tag{1}$$

$$u_1|_{S_1} = u_2|_{S_1} \tag{2}$$

$$\left.\frac{\partial u_2}{\partial n_2}\right|_{S_2} = 0 \tag{3}$$

$$u_2|_{S_2} = V \tag{4}$$

The inverse problem consists in determining the normal component J_n of the current J in the celebral cortex:

$$J_n = \sigma_1 \left.\frac{\partial u_1}{\partial n_1}\right|_{S_1} - \sigma_2 \left.\frac{\partial u_2}{\partial n_1}\right|_{S_1} \tag{5}$$

In order to find J_n, measurements of the electric potential registered on the surface of the skull are used. An approach to solving this problem is proposed and justified in [1–4]. Here we follow another approach based on the technique of spherical harmonics for processing the experimental electroencephalographic data [5]. We generalize this approach by taking into account the condition that the centers of spheres S_1 and S_2 are separated by the distance ε.

Let us choose the origin of coordinates at the center of the outer sphere S_2 and direct the z-axis through the center of the inner sphere S_1. The radius of the sphere S_1 can be expressed in terms of spherical coordinates θ, φ, $r = r(\theta, \varphi)$ as follows:

$$r = \sqrt{a^2 - \varepsilon^2 \sin^2\theta} + \varepsilon\cos\theta \tag{6}$$

Using spherical functions $Y_{lm}(\theta, \varphi)$, the electric potentials u_1 and u_2 in the brain and in the skull can be represented respectively as

$$u_1 = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} A^0_{lm} r^l Y_{lm}(\theta, \varphi) \tag{7}$$

$$u_2 = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \left(A_{lm} r^l + B_{lm} r^{-l-1}\right) Y_{lm}(\theta, \varphi) \tag{8}$$

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Equation (1) is satisfied in both the cases. The potential V on the surface of the sphere S_1 and the normal component J_n on the surface of the sphere S_2 are represented by the following formulas:

$$V = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} V_{lm} Y_{lm}(\theta, \varphi) \tag{9}$$

$$J_n = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} J_{lm} Y_{lm}(\theta, \varphi)$$

The coefficients V_{lm} can be calculated using experimental data. The unknown coefficients J_{lm} are expressed by V_{lm} from the above boundary conditions under the assumption that the distance ε between the centers of the spheres is so small that the terms of second and higher orders of smallness in ε can be ignored.

Substituting (6) – (8) into boundary conditions (2) – (4) and solving the resulting system under the above assumption on ε, we obtain the following set of expressions:

$$J_{00} = -\varepsilon \frac{2}{3} \sqrt{\frac{1}{3}} \left[2(\sigma_1 - \sigma_2) a^{-1} b^{-1} + (\sigma_1 + 2\sigma_2) a^{-4} b^2 \right] V_{10}$$

$$J_{1(-1)} = \left[(\sigma_1 - \sigma_2) \frac{2b^{-1}}{3} + (\sigma_1 + 2\sigma_2) a^{-3} \frac{b^2}{3} \right] V_{1(-1)} - \varepsilon \sqrt{\frac{3}{15}} \left[(\sigma_1 - \sigma_2) \frac{3b^{-2}}{5} + (6\sigma_1 + 9\sigma_2) a^{-5} \frac{2b^3}{5} \right] V_{2(-1)}$$

$$J_{10} = \left[(\sigma_1 - \sigma_2) \frac{2b^{-1}}{3} + (\sigma_1 + 2\sigma_2) a^{-3} \frac{b^2}{3} \right] V_{10} - \varepsilon \sqrt{\frac{4}{15}} \left[(\sigma_1 - \sigma_2) \frac{3b^{-2}}{5} + (6\sigma_1 + 9\sigma_2) a^{-5} \frac{2b^3}{5} \right] V_{20}$$

$$J_{11} = \left[(\sigma_1 - \sigma_2) \frac{2b^{-1}}{3} + (\sigma_1 + 2\sigma_2) a^{-3} \frac{b^2}{3} \right] V_{11} - \varepsilon \sqrt{\frac{3}{15}} \left[(\sigma_1 - \sigma_2) \frac{3b^{-2}}{5} + (6\sigma_1 + 9\sigma_2) a^{-5} \frac{2b^3}{5} \right] V_{21}$$

$$\begin{aligned} J_{l(-l)} = {} & \left[l(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+1) b^{-l}}{2l+1} + [l\sigma_1 + (l+1)\sigma_2] a^{-l-2} \frac{l b^{l+1}}{2l+1} \right] V_{l(-l)} \\ & + \varepsilon \sqrt{\frac{2l+1}{4(l+1)^2 - 1}} \left(l^2 - 2\right)(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+2) b^{-l-1}}{2l+3} V_{(l+1)(-l)} \\ & - \varepsilon \sqrt{\frac{2l+1}{4(l+1)^2 - 1}} \left[\left(l^2 + 3l + 2\right)\sigma_1 + (l+2)^2 \sigma_2 \right] a^{-l-4} \frac{(l+1) b^{l+2}}{2l+3} V_{(l+1)(-l)}, \quad l > 1 \end{aligned}$$

$$\begin{aligned} J_{ll} = {} & \left[l(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+1) b^{-l}}{2l+1} + [l\sigma_1 + (l+1)\sigma_2] a^{-l-2} \frac{l b^{l+1}}{2l+1} \right] V_{ll} \\ & + \varepsilon \sqrt{\frac{2l+1}{4(l+1)^2 - 1}} \left(l^2 - 2\right)(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+2) b^{-l-1}}{2l+3} V_{(l+1)l} \\ & - \varepsilon \sqrt{\frac{2l+1}{4(l+1)^2 - 1}} \left[\left(l^2 + 3l + 2\right)\sigma_1 + (l+2)^2 \sigma_2 \right] a^{-l-4} \frac{(l+1) b^{l+2}}{2l+3} V_{(l+1)l}, \quad l > 1 \end{aligned}$$

$$\begin{aligned} J_{lm} = {} & \left[l(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+1) b^{-l}}{2l+1} + [l\sigma_1 + (l+1)\sigma_2] a^{-l-2} \frac{l b^{l+1}}{2l+1} \right] V_{lm} \\ & + \varepsilon \sqrt{\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1}} \left(l^2 - 2\right)(\sigma_1 - \sigma_2) a^{l-1} \frac{(l+2) b^{-l-1}}{2l+3} V_{(l+1)m} \\ & - \varepsilon \sqrt{\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1}} \left[\left(l^2 + 3l + 2\right)\sigma_1 + (l+2)^2 \sigma_2 \right] a^{-l-4} \frac{(l+1) b^{l+2}}{2l+3} V_{(l+1)m} \\ & + \varepsilon \sqrt{\frac{l^2 - m^2}{4l^2 - 1}} (l-1)^2 (\sigma_1 - \sigma_2) a^{l-3} \frac{l b^{-l+1}}{2l-1} V_{(l-1)m} \\ & - \varepsilon \sqrt{\frac{l^2 - m^2}{4l^2 - 1}} \left[\left(l^2 + l - 1\right)\sigma_1 + \left(l^2 + 2l - 1\right)\sigma_2 \right] a^{-l-2} \frac{(l-1) b^{l}}{2l-1} V_{(l-1)m}, \quad l > m > 1 \end{aligned}$$

In order to determine J_n numerically, we substitute the values $\{V_i\}$ of the potential V measured at chosen points $P_i = \{\theta_i, \varphi_i\}$ on the surface of the sphere S_2 into the formula

$$V(\theta, \varphi) = \sum_{l=0}^{N} \sum_{m=-l}^{l} V_{lm} Y_{lm}(\theta, \varphi) \tag{10}$$

and finally obtain the system of equations

$$V_i = \sum_{l=0}^{N} \sum_{m=-l}^{l} V_{lm} Y_{lm}(\theta_i, \varphi_i), \quad i = 1, \ldots, p \tag{11}$$

for the unknown $\{V_{lm}\}$ at each point. Formula (10) is a particular case of formula (9) with a finite upper limit N in the first sum. The value of N in (11) is taken in such a way that the number of equations should be equal to the number of unknowns. After solving system (11), the approximate values $\widetilde{V}_{lm}$ of V_{lm} are substituted into formula (10) to obtain approximate values of the potential V at any point on the surface of the sphere S_2. To calculate J_n, we use $\widetilde{V}_{lm}$ instead of V_{lm} in the set of the expressions given above.

Another algorithm for calculating the needed approximate values $\widetilde{V}_{lm}$ can be proposed. By $V_k(\theta, \varphi)$, $k = 1, \ldots, N$, we denote the solution to system (11) for $V_i = 1$ if $i = k$ and $V_i = 0$ if $i \neq k$. Knowing the values $\{V_k\}$ at the points $\{P_k\}$ of electroencephalographic measurements, we can represent the approximate potential V in the form

$$V(\theta, \varphi) = \sum_{k=1}^{N} V_k \cdot V_k(\theta, \varphi)$$

A number of numerical experiments were performed with the use of electroencephalographic measurements taken from [6, 7]. The results of our experiments confirm the above consideration.

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